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Statistical properties of conductance through a quantum dot in the Coulomb blockade regime

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Abstract

We investigate the statistical properties of the conductance through a quantum dot in the Coulomb blockade regime. By taking into account the charging energy we calculate the correlation function of conductance by the use of two types of energy level distributions of the dot, the Poisson distribution and the Wigner–Dyson distribution. In both cases, the conduction correlation obtained as a function of the difference in occupation number shows similar behaviour, decaying in the range where occupation differences are small, in agreement with experiment, but exhibits different tail behaviour where occupation differences are large.

1. Introduction

In the transport of electrons through a quantum dot, there are two basic physical ingredients which play an important role: resonant tunnelling and Coulomb blockade. The levels of the dot provide the bridge for the tunnelling so that the electrons in energies coincident with the levels can tunnel easily from one electrode to the other. Due to the Coulomb blockade, however, the resonant levels are shifted by the interaction. In the Coulomb blockade regime the conductance exhibits alternate peaks and valleys by sweeping the gate voltage which tunes the position of the levels. The spacing of the peaks depends on the energies of the resonant levels of the dot plus the charging energy which is needed to overcome the Coulomb blockade. Since the appearance of discrete energy levels of the dot is a result of quantum interference determined by the size, shape, impurities, etc, the conductance will fluctuate if these factors are varied. By applying a magnetic field to the dot and changing its strength over a range one can obtain detailed information on the fluctuations of the dot because the additional phase changes of the wavefunctions due to the magnetic field depend on the details of the randomness on the dot and will shift the positions of the levels. In this way the mesoscopic fluctuations of the elastic tunnelling in the valleys of the conductance is measured in [1] and studied theoretically in [2]. The transient current of a quantum dot in the Coulomb blockade regime is also measured to investigate the relaxation rate of spin and charge [3].

In this paper we investigate the fluctuations of the conductance of a quantum dot due to the random distribution of energy levels. The effect of the Coulomb blockade is taken into account by using many-body states of the dot in an equivalent network for the Schrödinger equation. We consider a quantum dot as weakly coupled to reservoirs. There are many levels on the dot corresponding to a sequence of resonant peaks of the conductance in sweeping the gate voltage. In the valleys the conductance is strongly suppressed due to the charging energy which is needed to add electrons to the dot. However, an electron can tunnel through the dot via a large number of virtual dot states, resulting in a residual conductance in the valleys. This tunnelling process in the valleys is known as co-tunnelling [4] and may reflect some information about the statistics of dot levels. We calculate the correlation function of the conductance in the valleys and peaks by the use of two types of nergy level distribution, the Poisson distribution and the Wigner–Dyson distribution. In both cases the conduction correlation obtained as a function of the difference in the occupation number shows similar behaviour decaying in the range of small occupation differences, in agreement with experiment, but the valley–valley correlation exhibits different tail oscillations for large occupation differences, reflecting the sensitivity to the level statistics.

In the next section we describe the used model and the basic formula. In section 3 we present numerical results and compare them with the experiments. In the last section we give a brief summary of conclusions.

2. The model and formula

We consider a quantum dot embedded between two reservoirs. We suppose that the reservoirs have only one channel and can be described with a one-dimensional tight-binding model. The Hamiltonian of the dot coupled to the reservoirs can be modelled as [5]

$$H = H^{\mathrm{R}} + H^{\mathrm{QD}} + H^{\mathrm{T}} \tag{1}$$

where H^{QD} is the sub-Hamiltonian of electrons on the isolated dot, H^R describes the motion of electrons in the left and right reservoirs and H^T represents the coupling between dot and reservoirs. The sub-Hamiltonians can be written as

$$H^{\mathrm{R}} = \sum_{m\neq -1,0} t_0(a_m^\dagger a_{m+1} + a_{m+1}^\dagger a_m) + \sum_{m\neq 0} \epsilon_0 a_m^\dagger a_m \tag{2}$$

$$H^{\mathrm{QD}} = \sum_{i=1}^{N}(\xi_i + V)d_i^\dagger d_i + \frac{e^2}{2C}\left(\sum_{i=1}^{N} d_i^\dagger d_i\right)^2 \tag{3}$$

$$H^{\mathrm{T}} = \sum_{i=1}^{N}\left(t_i^{(\mathrm{L})} a_{-1}^\dagger d_i + t_i^{(\mathrm{R})} a_1^\dagger d_i + t_i^{(\mathrm{L})*} d_i^\dagger a_{-1} + t_i^{(\mathrm{R})*} d_i^\dagger a_1\right) \tag{4}$$

where a_m and d_i are annihilation operators of electrons on the mth site of the reservoirs and the ith bound state of the close dot, respectively, $t^{\mathrm{L(R)}}$ is the coupling of a dot state to the contact site of the left (right) reservoir, and N is the number of bound states. Here, the electrons on the close dot are characterized by the level energy ξ_i, the dot potential V induced by the gate voltage, and the charging energy in an effective capacitance C. In reservoirs the electrons have no interaction and are described by site energy ϵ_0 and hopping t_0. The chemical potential of reservoirs is set to be the zero of energy. On the dot, we use a simple model with constant interaction in which the total energy due to the electron–electron interaction solely depends on the number of electrons on the dot. Such a charging energy is given by $e^2/2C$ invoking the electrostatic energy of a classical capacitance C. In this paper we are interested only in the statistics of levels and omit the spin index of the electrons. In the following we calculate transport properties of electrons by extending the method of the equivalent many-body state network proposed in [5] to the case of a disordered dot. We neglect the inelastic processes and those processes that contain more than one electron in the leads. Thus the calculations are still approximate. In particular, to deal with the non-equilibrium we assume local equilibrium on dot, in the left and right leads, and use an approximate Landauer-like formula to calculate the conductance in the linear-response regime. The disorder should be understood as caused by randomly distributed scatterers, such as impurities and defects, or by shape irregularities. In all cases the statistical properties of single-particle levels could be described by random matrix theory (RMT) (for a review see [6]). Here we follow the theory of Kubo who postulated that the levels are randomly distributed. The details of the level spectrum of an individual dot would be determined by its precise geometry and the spatial locations of the atoms. Other dot samples with similar size may have a different spectrum due to different shapes and atom locations even though they have the same average level spacing $\overline{\Delta}$. In the statistical sense one should establish a probability function $P(\Delta)$ which specifies the distribution of the level spacing of all the samples in a given ensemble. Typically, the Poisson and Wigner–Dyson distributions are used. The former corresponds to the 'disordered insulator' and non-chaotic level statistics, while the latter describes the properties of a 'disordered metal' and chaotic features.

We consider the transmission of a single electron through the dot which has M electrons in levels below the chemical potential. By adding the tunnelling electron there are totally $M+1$ electrons in the relevant many-body states which are subjected to a constraint that there is at most only one electron in the leads [5]. These states form a sub-Hilbert space, and we use the following many-body wavefunctions as the basis

$$\Phi_{m,L} = a_m^\dagger \left\{ \prod_{i\in L} d_i^\dagger \right\} |0\rangle \tag{5}$$

$$\Phi_{L'} = \left\{ \prod_{i\in L'} d_i^\dagger \right\} |0\rangle \tag{6}$$

where L and $L^{'}$ are sets of M and $M+1$ single-particle states of the close dot, respectively, and $|0\rangle$ denotes the vacuum. A state in this subspace can be expressed as a liner combination of basis functions

$$\Psi = \sum_{m\neq 0} \sum_{L} p_{m,L} \Phi_{m,L} + \sum_{L'} q_{L'} \Phi_{L'}. \tag{7}$$

By applying the Hamiltonian on Ψ one can get equations for coefficients $p_{m,L}$ and $q_{L'}$. By solving these equations we obtain the transmission amplitude through the dot as

$$t(\epsilon, V) = \frac{-2\mathrm{i}t_0 S_{\mathrm{LR}} \sin k}{(t_0\mathrm{e}^{-\mathrm{i}k} - S_{\mathrm{LL}})(t_0\mathrm{e}^{-\mathrm{i}k} - S_{\mathrm{RR}}) - |S_{\mathrm{RL}}|^2} \tag{8}$$

where

$$S_{\lambda\lambda'} = \sum_{i=M_1+1}^{N} \frac{t_i^{(\lambda)*} t_i^{(\lambda')*}}{\epsilon - \xi_i - V - (2M_1+1)\mathrm{e}^2/2C} \tag{9}$$

$$\epsilon = 2t_0 \cos k + \epsilon_0 \tag{10}$$

with ϵ being the energy of the tunnelling electron which is regarded as free in reservoirs and M_1 labelling the lowest level occupied by electrons. When the temperature is non-zero, there are many channels contributing to transport properties. If we only consider the elastic scattering and all the states in the dot are not degenerate, these channels are independent and the contribution from each channel is the corresponding transmission coefficient multiplying a statistical weight determined by the thermal probability. If the average level spacing $\overline{\Delta}$ is in the order of k_BT with T the temperature and k_B the Boltzman constant, we only need to take into account several low exciting states. The statistical weight of the jth channel can be written as

$$F_j(T) = \frac{\exp(E_j/k_BT)}{\sum_i \exp(E_i/k_BT)} \tag{11}$$

where E_j is the total energy of the many-body state in channel j, and the sum in the denominator is over all the independent channels. We can calculate the conductance at temperature T from the transmission coefficient obtained by using the Landauer–Bütticker formula:

$$G(V,T) = -\frac{e^2}{h}\int \mathrm{d}\epsilon \left[\sum_j F_j(T)|t_j(\epsilon,V)|^2\right]\frac{\partial f}{\partial \epsilon} \tag{12}$$

where f is the Fermi distribution of free electrons in the reservoirs and $t_j(\epsilon, V)$ is the transmission amplitude of channel j. From this formula the conductance in the linear regime with infinitesimal bias voltage is obtained. In the presence of interactions, it is valid at zero temperature. At finite temperature, however, the approximation is well controllable if the thermal excitation energy is much smaller than the charging energy ($k_BT \ll e^2/2C$) so that the single-particle tunnelling picture is not much disturbed by the thermal excitations. As the obtained $G(V, T)$ depends on the gate voltage V, the correlation function of the linear conductance can be calculated as

$$C(V_1, V_2) = \frac{\langle G(V_1)G(V_2)\rangle - \langle G(V_1)\rangle\langle G(V_2)\rangle}{\sqrt{\mathrm{var}\, G(V_1)}\sqrt{\mathrm{var}\, G(V_2)}} \tag{13}$$

where V_1 and V_2 are gate voltages. The averaging is taken over the distribution probability of the ensemble. In the next section we will use both the Poisson and Wigner–Dyson distributions and compare the obtained results with experiments.

Equation (12) could be extended to calculate the current in finite bias if we assume that the dot is in local equilibrium with the Fermi level situated in the middle between Fermi levels of the left and right reservoirs and neglect all inelastic processes. In this way the electric current is approximately expressed as

$$I(V_b) = \frac{e}{h}\int \mathrm{d}\epsilon \left[\sum_j F_j(T)|t_j(\epsilon,V_b)|^2\right]\left[f\left(\epsilon - \frac{eV_b}{2}\right) - f\left(\epsilon + \frac{eV_b}{2}\right)\right] \tag{14}$$

where V_b is the bias voltage applied between two reservoirs. As pointed by Meir and Wingreen [7], the calculation of the current in a relatively large bias at finite temperature in the presence of interaction is a highly non-trivial problem, and should include a lot of excitations which are hardly accounted for. In order to estimate the fluctuations of current due to disorder and other disturbances of physical factors which are not taken into account in equation (14), we calculate the variance of current as

$$\mathrm{var}\, I(V_b) = \langle I(V_b)^2\rangle - \langle I(V_b)\rangle^2 \tag{15}$$

where $\langle\ldots\rangle$ denotes the ensemble averaging. This result is not used to compare with experiments, but it can be used to estimate how the fluctuations of current grow with increasing bias voltage.

3. Numerical results

Firstly we calculate transport properties by using the Poisson distribution of level spacing

$$P(\Delta) = \frac{1}{\overline{\Delta}} e^{-\Delta/\overline{\Delta}}. \tag{16}$$

where $P(\Delta)\mathrm{d}\Delta$ is the probability of finding the spacing of the adjacent levels in the interval $(\Delta,\ \Delta + \mathrm{d}\Delta)$. In this distribution, the levels on the dot behave as independent random variables. By using the formula given in the previous section we can calculate peak–peak, peak–valley and valley–valley correlation functions for conductance and fluctuations of current. The results for the correlation of conductance are shown in figure 1. Here N_1 and N_2 are the occupation numbers of the dot, corresponding to gate voltages V_1 and V_2 in equation (13), respectively. It can be seen that valley–valley, peak–valley and peak–peak correlation functions decay similarly as the difference of the occupation numbers increases in the range $N_2 - N_1 < 4$. The decay rate is slightly different for these curves. The valley–valley correlation function decays most slowly. The peak–peak correlation function reaches a negative value in this range. In the experiments of [1], the average is taken over the applied magnetic fields. The effect of the fields is to change the quantum interference of states within the dot. Thus, for the dot with irregularities, sweeping the field will cause variations of dot levels which reflect the nature of disorder of the dot. In comparison with figure 4(c) of [1], where only the correlation functions in range $N_2 - N_1 < 4$ are shown, we find that the calculated results are generally in good agreement with the experimental data, except that the calculated correlations decrease a little more quickly than the experimental ones. This may stem from the high probability for the small level spacing in the Poisson distribution. In the range $N_2 - N_1 > 4$, an apparent difference of the peak–peak curve from the others appears. The valley–valley and peak–valley correlation functions exhibit small fluctuations near zero, while the peak–peak correlation increases from a negative value and approaches to zero. The transport at the peaks is produced by the resonant tunnelling and the positions of peaks are mainly determined by the Coulomb charging energy which is almost regular and independent of the randomness of the dot. This results in the slow decay of the peak–peak correlation. At the same time, the transport at the valleys is caused by the co-tunnelling, which is related to a group of dot levels and is influenced by the level statistics and randomness. For the Poisson distribution the levels are regarded as independent random variables, so the valley–valley and peak–valley correlations approach zero more rapidly. Here the ensemble average is taken for the distribution of levels.

We also calculate fluctuations of electric current var I. The result are plotted in figure 2. It can be seen from equation (14) that, by increasing the absolute value of the bias voltage, the transmission energy window is enlarged to include more resonant levels. This results in a monotonic increase of current. However, the intensity of the fluctuations of current does not increase monotonically with bias voltage, as can be seen in figure 2. There are peaks with oscillating heights. The narrow peaks in figure 2 correspond to fluctuations of the contribution from the new resonant level which is added to the energy window for increasing $|V_\mathrm{b}|$. The wider peaks between the narrow peaks represent current fluctuations in the co-tunnelling channels, whose contributions are added to the window. The intensity of the narrow peaks is decreased by enlarging the window because the summing over more contributions from resonant levels suppresses fluctuations of the total current. However, flu!lactuations of the co-tunnelling are enhanced as more co-tunnelling channels are participating in the transport on enlarging the window. However, when the window is large enough, there are so many resonant levels that the difference in fluctuations between the resonant tunnelling and the co-tunnelling disappears. This result could be used to estimate how the fluctuations of current grow when

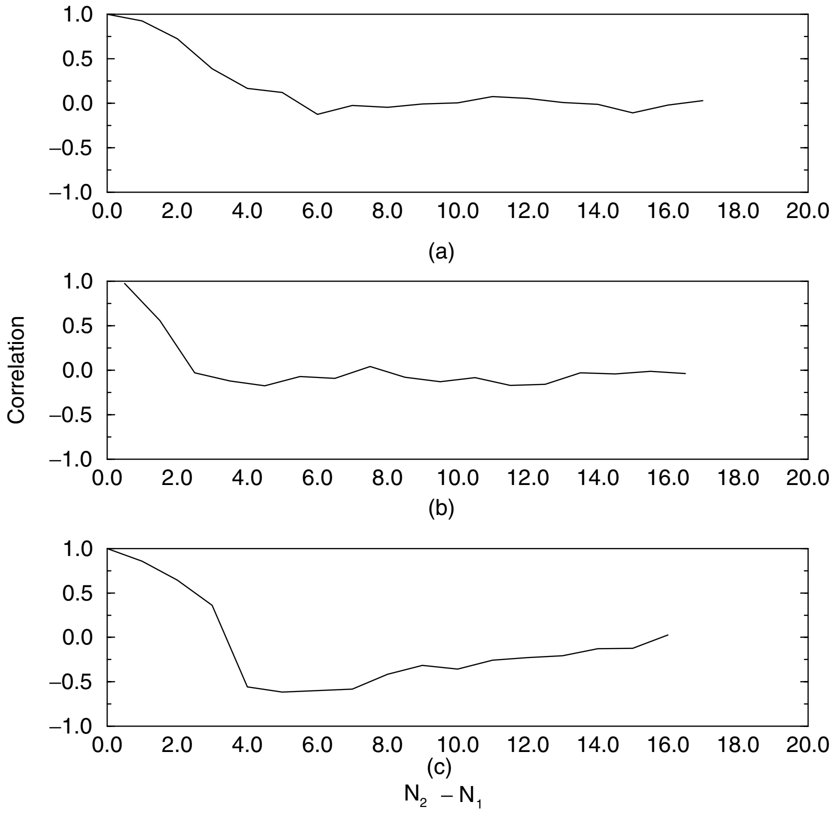

Figure 1. The correlation function of conductance $C(N_1, N_2)$ versus $N_2 - N_1$ for the Poisson distribution of the level spacing, where N_1 and N_2 are the occupation numbers of the dot corresponding to gate voltages V_1 and V_2 in equation (13), respectively. (a) The valley–valley correlation function. The average level spacing $\overline{\Delta} = 0.000\,0088t_0$. $e^2/2C = 0.00033t_0$. $k_BT = 0.000\,0132t_0$. $t = 0.005t_0$. (b) The peak–valley correlation function. $\overline{\Delta} = 0.000\,0088t_0$. $e^2/2C = 0.000\,33t_0$. $k_BT = 0.000\,0088t_0$. $t = 0.005t_0$. (c) The peak–peak correlation function. $\overline{\Delta} = 0.000\,0132t_0$. $e^2/2C = 0.000\,33t_0$. $k_BT = 0.000\,0088t_0$. $t = 0.005t_0$.

the bias voltage is increased. The non-monotonic behaviour of the intensity is due to the very weak coupling between the dot and the reservoirs ($t \ll t_0$), which leads to small transmission amplitudes, so that the tunnelling of electrons can be viewed as one by one even in a finite bias voltage.

It is possible that quantum chaos appears in the presence of interactions among particles in a system [8–10]. From the classical point of view, in the absence of the interaction, the energy of every single particle is a constant of motion, and therefore the problem is integrable. The inclusion of the Coulomb interactions breaks the conservation of the energies of the single particles and leads to an irregular dynamics in a confined system. As a result the trajectory of a particle in the phase space may cover all parts of space. From the viewpoint of quantum mechanics this situation corresponds to the extended state in a disordered metal, and the distribution of the level spacing is characteristic of a Wigner–Dyson function

$$P(\Delta) = \frac{\pi\Delta}{2\overline{\Delta}^2}\exp\left(-\frac{\pi\Delta^2}{4\overline{\Delta}^2}\right) \qquad (17)$$

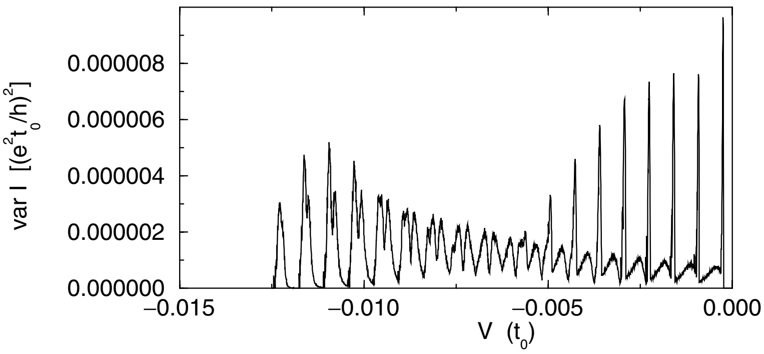

Figure 2. The variance of the electric current as a function of the bias voltage. The gate voltage is zero. The other parameters are the same as those in figure 1(a). Here only the data for $V_b < 0$ are shown.

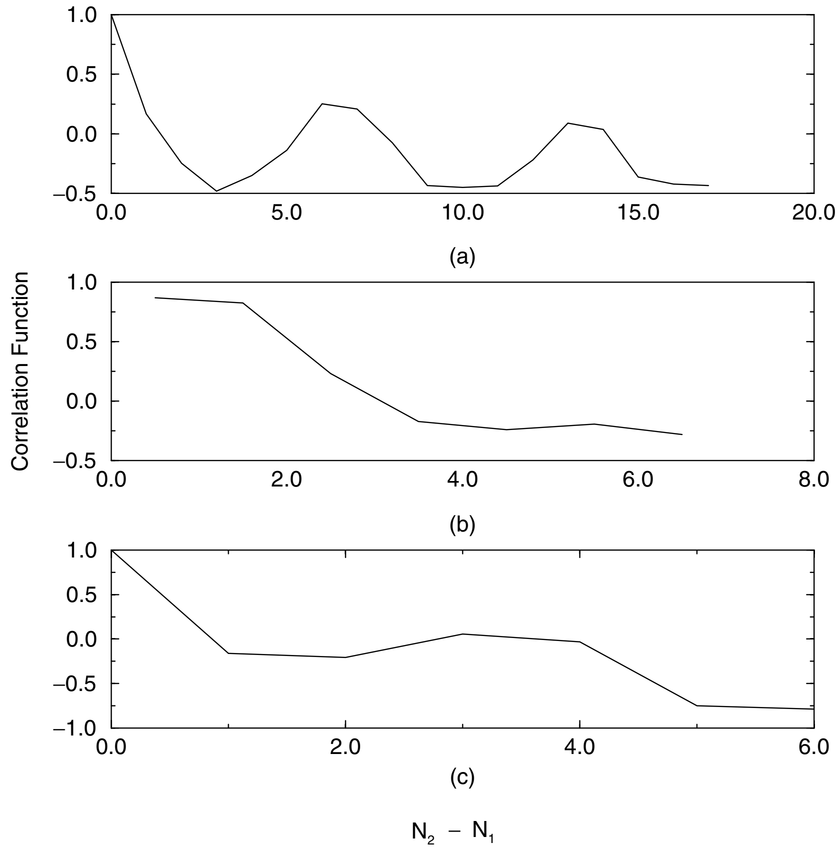

Figure 3. The correlation function of the conductance $C(N_1, N_2)$ plotted against $N_2 - N_1$ for a Poisson distribution of the level spacings. The parameters are the same as those used in figure 1. (a) The valley–valley correlation function. (b) The peak–valley correlation function. (c) The peak–peak correlation function.

where $\overline{\Delta}$ is the average level spacing as used in the Poisson distribution. Adopting the Wigner–Dyson distribution, we calculate the correlation function of conductance. The results are shown in figure 3. It can be seen that in the range of $N_2 - N_1 < 4$ the general behaviour of the curves is only slightly changed from that of the Poisson distribution. In range $N_2 - N_1 > 4$ an apparent deviation appears in the range $N_2 - N_1 > 4$, where the valley–valley correlation shows long-tail oscillations. Since the valley–valley correlation reflects information about the levels participating in the co-tunnelling, such long-tail fluctuations can be regarded as a characteristic of the Wigner–Dyson distribution. In fact, it reflects the intrinsic correlation among levels in the Wigner–Dyson distribution. In the Poisson distribution the levels are nearly independent random variables, so there are no long-tail oscillations in figure 1.

4. Conclusions

In this paper we have presented calculations of fluctuations of current and correlation functions of conductance in a quantum dot by taking into account both the Coulomb blockade effect and the distribution of dot levels. We have obtained numerical results by the use of two types of distribution for the energy levels on the dot, the Poisson distribution and Wigner–Dyson distribution. The correlation functions in both cases show similar behaviour: decaying in the range of small differences in the occupation number, in agreement with experiment, but the valley–valley correlation exhibits distinguishing long-tail oscillations for large occupation differences in theWigner–Dyson distribution. The results for current fluctuations indicate different behaviour between the resonant peaks and valleys. For the former, the contribution to the transport is mainly from one resonant level. For the latter, more levels participate in the tunnelling, thus properties are more closely related to the level statistics on the dot.

Acknowledgments

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